Solution $Q_3$ EE 303

EE 303: Model Answer of Q3:-

Midterm I January $3^{th}$, 2012

Q3:- Given the following system of $Eq_s$

$$-3y + z = 16.1$$
$$2x + 6y - 4z = -49$$
$$-8x - 2y + 5z = 18.9$$

a) Use Forward Gauss elimination with partial pivoting to solve the system.

b) Solve the system using LU-decomposition

Solution

The augmented matrix is

$$\begin{bmatrix} 0 & -3 & 1 & 16.1 \\ 2 & 6 & -4 & -49 \\ -8 & -2 & 5 & 18.9 \end{bmatrix}$$

with partial pivoting: Interchangin $R_1$ & $R_3$

$$\begin{bmatrix} -8 & -2 & 5 & 18.9 \\ 2 & 6 & -4 & -49 \\ 0 & -3 & 1 & 16.1 \end{bmatrix}$$

To eliminat $x$ from $R_2$ & $R_3$

$$f_{21} = \frac{2}{-8} = -0.25 \quad , \quad f_{31} = 0$$

$R_2' \leftarrow R_2 - f_{21} \cdot R_1$ , $R_3' \leftarrow R_3 - f_{31} \cdot R_1$

$$\begin{bmatrix} -8 & -2 & 5 & 18.9 \\ 0 & 5.5 & -2.75 & -44.275 \\ 0 & -3 & 1 & 16.1 \end{bmatrix}$$

$$f_{32} = \frac{-3}{5.5} = -0.5455$$

$R_3'' \leftarrow R_3' - f_{32} \cdot R_2$

$$\begin{bmatrix} -8 & -2 & 5 & 18.9 \\ 0 & 5.5 & -2.75 & -44.275 \\ 0 & 0 & -0.5 & -8.052 \end{bmatrix}$$

$$-0.5z = -8.052 \Longrightarrow \boxed{z = 16.104}$$

Backsubstitution

$$5.5y - 2.75z = -44.275 \Rightarrow y = \frac{-44.275 + 2.75(16.104)}{5.5}$$

$$\therefore \boxed{y = 0.002}$$

$$-8x - 2y + 5z = 18.9 \Rightarrow x = \frac{5z - 2y - 18.9}{8} = \frac{5(16.104) - 2(0.002) - 18.9}{8}$$

$$\boxed{\therefore x = 7.702}$$

substitute in the original Eqs

$1^{st}$ Eq: $\underbrace{-3(0.002) + 16.104}_{L.H.S} \Rightarrow \boxed{L.H.S = 16.09}$

Absolute Error $= |R.H.S - L.H.S| = |16.1 - 16.09|$

$$\boxed{E = 0.01}$$

2nd Eq:- $L.H.S = 2(7.702) + 6(0.002) - 4(16.104)$

$\therefore L.H.S = -49 = R.S.H$ #

$\boxed{\text{Error} = 0}$

Eq:- $L.H.S = -8(7.702) - 2(0.002) + 5(16.104)$

$\therefore L.S.H = 18.9 = R.H.S$

$\boxed{\text{Error} = 0}$

) $\because AX = B$ , $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ , $B = \begin{bmatrix} 16.1 \\ -49 \\ 18.9 \end{bmatrix}$

$A = LU$

$L \cdot U = A$

The L_Matrix is the matrix of eliminating factors obtained in section (a)

U_Matrix is the matrix of the Gauss elimination without the forth column

$$L = \begin{bmatrix} 1 & 0 & 0 \\ f_{21} & 1 & 0 \\ f_{31} & f_{32} & 1 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ 0 & -0.5455 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix} \Rightarrow U = \begin{bmatrix} -8 & -2 & 5 \\ 0 & 5.5 & -2.75 \\ 0 & 0 & -0.5 \end{bmatrix}$$

$L(UX) = B \Rightarrow L.Z = B$ , $Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 18.9 \\ -49.0 \\ 16.1 \end{bmatrix}$$

$\boxed{\therefore z_1 = 18.9}$

$l_{21} \cdot z_1 + z_2 = -49 \Rightarrow z_2 = -49 - 18.9 \times (-0.25)$

$\boxed{\therefore z_2 = -44.275}$

$l_{31} \cdot z_1 + l_{32} z_2 + z_3 = 16.1 \Rightarrow z_3 = 16.1 - (0 \times z_1) - (-0.5455 \times -44.275)$

$\boxed{\therefore z_3 = -8.052}$

$$\therefore UX = Z \Rightarrow \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18.9 \\ -44.275 \\ -8.052 \end{bmatrix}$$

$u_{33} \cdot z = -8.052 \Rightarrow z = \dfrac{-8.052}{-0.5} \Rightarrow \boxed{z = 16.104}$

$u_{22} y + u_{23} z = -44.275$

$\therefore y = \dfrac{-44.275 + 2.75 \times 16.104}{5.5} \Rightarrow \boxed{y = 0.002}$

$u_{11} \cdot x + u_{12} y + u_{13} z = 18.9$

$$x = \frac{18.9 - (-2)(0.002) - 5(16.104)}{-8}$$

$\boxed{\therefore x = 7.702}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7.702 \\ 0.002 \\ 16.104 \end{bmatrix}$$

The same results on section (a)

```c
//This program will use the bisection(Half-interval) method to
//find the root of any non-linear equation
//provided that the root is inside the interval[a,b]
//Dr. Idris El-Feghi
//EE 303 Numerical Techniques and Peogramming
//-------------------------------------------------------------------------//
#include<stdio.h>
#include<math.h>
float f(float);
void main()
{
        float a,b,c;
        scanf("%f   %f",&a,&b);
        c=(a+b)/2.;
        if(f(a)*f(b)<0)
             {
                 while(fabs(f(c))>0.0000001)
                 {
                        if(f(a)*f(c)<0)
                             b=c;
                        else
                             a=c;
                           c=(a+b)/2.0;
                           printf("\n%f   %f  %15.9f %15.9f",a,b,c,f(c));
                 }

             }
        else
             printf("\nthere is no solution");
 }
 float f(float x)
 {
 float ret_Value;
 //ret_Value=exp(x)-sin(x);
 ret_Value=2*sin(x)-exp(x/4)-1;
 return(ret_Value);
 }
```

Using the general Formula:

$$L.U = A \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$U_{11} = a_{11}, \quad U_{12} = a_{12}, \quad U_{13} = a_{13}$

$\cancel{l_{21} . U_{11} + U}$

$l_{21} . U_{11} = a_{21} \Rightarrow \boxed{l_{21} = \frac{a_{21}}{U_{11}}} \rightarrow U_{11} = a_{11}$ should not be zero.

$l_{21} U_{12} + U_{22} = a_{22} \Rightarrow \boxed{U_{22} = a_{22} - l_{21} . U_{12}}$

$l_{13} + U_{23} = a_{23} \Rightarrow \boxed{U_{23} = a_{23} - l_{21} . U_{13}}$

$l_{31} . U_{11} = a_{31} \Rightarrow \boxed{l_{31} = \frac{a_{31}}{U_{11}}}$

$l_{31} . U_{12} + l_{32} . U_{22} = a_{32} \Rightarrow \boxed{l_{32} = \frac{a_{32} - l_{31} U_{12}}{U_{22}}}$, $U_{12}$ should not be equal to zer if it so interchange Rows

$l_{31} . U_{13} + l_{32} . U_{23} + U_{33} = a_{33}$

$\therefore \boxed{U_{33} = a_{33} - l_{31} . U_{13} - l_{32} . U_{23}}$

Ex: $A = \begin{bmatrix} 0 & -3 & 1 \\ 2 & 6 & -4 \\ -8 & -2 & 5 \end{bmatrix}$, Applying Pivoting

$$A = \begin{bmatrix} -8 & -2 & 5 \\ 2 & 6 & -4 \\ 0 & -3 & 1 \end{bmatrix}$$

$a_{11} = -8, \; a_{12} = -2, \; a_{13} = 5$
$a_{21} = 2, \; a_{22} = 6, \; a_{23} = -4$
$a_{31} = 0, \; a_{32} = -3, \; a_{33} = 1$

$\therefore U_{11} = -8, \; U_{12} = -2, \; U_{13} = 5$

$$\boxed{l_{21} = \frac{2}{-8} = -0.25}$$

$$l_{22} = 6 - (-0.25 \times -2) = 6 - 0.5$$

$$\boxed{u_{22} = 5.5}$$

$$l_{23} = -4 - (-0.25 \times 5) = -4 + 1.25$$

$$\boxed{u_{23} = -2.75}$$

$$_{31} = \frac{0}{-8} \Rightarrow \boxed{l_{31} = 0}$$

$$_{32} = \frac{-3 - (0 \times -2)}{5.5} \Rightarrow \boxed{l_{32} = -0.5455}$$

$$_{33} = 1 - 0 - (-0.5455) \times (-2.75)$$

$$\therefore \boxed{u_{33} = -0.5}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ 0 & -0.5455 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} -8 & -2 & 5 \\ 0 & 5.5 & -2.75 \\ 0 & 0 & -0.5 \end{bmatrix}$$

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